

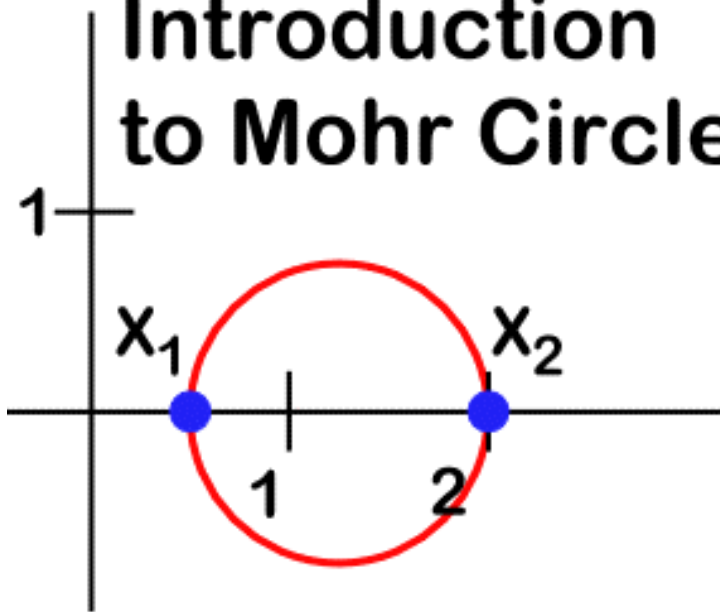


Online Introduction to Mohr Circles Visualizations of Fundamental Relationships

Please choose a topic then press the Go button.

-- Andy R. Bobyarchick --
-- University of North Carolina at Charlotte --

An Online Introduction to Mohr Circles



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Introduction

Welcome to an Online Introduction to Mohr Circles!

This guide is based on a workbook created by Win Means for a workshop he taught at the Geological Society of America annual meeting in 1992. That workbook is an essential element in the toolbox of any geologist interested in the visualization and analysis of stress and strain.

The OIMC is not intended to be a replacement for the workbook. Rather, it is a graphical presentation and summary of only a small part of the material contained in the workbook. The emphasis of the OIMC is animations that illustrate the progressive development of Mohr circle geometry as it is related to geographic space. The present version of the OIMC centers on the deformation tensor and strain.

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Andy R. Bobyarchick
Department of Geography
and Earth Sciences
University of North
Carolina at Charlotte
Charlotte, NC 28223
USA
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Internal Angles in the Mohr Circle

Given a deformation tensor D_{ij}

$$D_{ij} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

and the Mohr circle drawn by plotting

$$\begin{aligned} \mathbf{x}_1 &= (D_{11}, -D_{21}) \\ \mathbf{x}_2 &= (D_{22}, D_{12}) \end{aligned}$$

many uses of the construction (and those drawn from other tensors such as the stress tensor or the velocity gradients tensor) require a determination of the angles between two points on the circle. For example, the angle between transformed coordinates \mathbf{x}_i' produced by geographic coordinate transformations and \mathbf{x}_i is always 90° when measured from the pole P of the diagram. (In fact, that angle is always 90° regardless of the reference point used for measurement.)

The angle between any two points on a Mohr circle as measured from any third reference point on the circle is another invariant property of the Mohr diagram; this property is derived directly from circle geometry. In his workbook, Means referred to this "amazing" property as the "circular theatre rule."

Visualization

The animation cycles through a complete 360° rotation of an arbitrary reference point around the Mohr circle for a general deformation tensor. The point \mathbf{x}_1' is the position of the \mathbf{x}_1 tensor component after a 35° counterclockwise transformation of the geographic reference frame.

Observe that the reference angle between these two points on the Mohr circle does not vary. The angle between chords would be 35° regardless of the reference point used; the animation demonstrates this relationship.

Step incrementally through the part of the animation that takes the reference point between \mathbf{x}_1 and \mathbf{x}_1' . What happens here? Is the angle still invariant?

Summary

The angle between any two points on a Mohr circle, as measured from a third reference point, is invariant. The rotational part of a coordinate transformation can be read directly if a point and its transform can be located in the construction.

Components of the Tensor

Given a deformation tensor \mathbf{D}

$$\mathbf{D}_{ij} = \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the geographic coordinates of a particle

$$\mathbf{X}_i = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

the geographic coordinates of the particle in the deformed state are

$$\mathbf{x}_i = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} \cdot \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the standard equations for two-dimensional homogeneous deformation are then

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{11} \mathbf{X}_1 + \mathbf{D}_{12} \mathbf{X}_2 \\ \mathbf{x}_2 &= \mathbf{D}_{21} \mathbf{X}_1 + \mathbf{D}_{22} \mathbf{X}_2 . \end{aligned}$$

If the initial position vectors are

$$\begin{aligned} \mathbf{X}_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \mathbf{X}_2 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$

then

$$\mathbf{x}_i = \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the linear equations for the (1,0) corner of our unit square become

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{11} \\ \mathbf{x}_2 &= \mathbf{D}_{21} \mathbf{X}_1 . \end{aligned}$$

In the same way, the equations for the (0,1) corner become

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{12} \\ \mathbf{x}_2 &= \mathbf{D}_{22} . \end{aligned}$$

Visualization

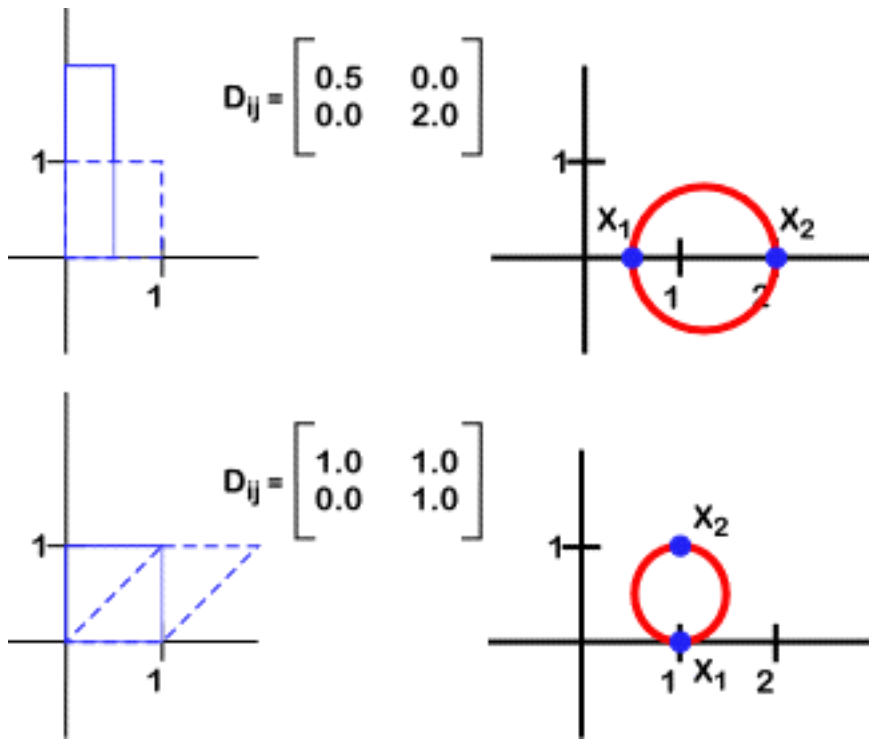
The \mathbf{x}_i vectors are plotted directly in geographic space. This animation represents a general deformation tensor and how it affects a unit square. Follow the progressive deformation by tracking the tensor components as they are derived from the geographic coordinates of the (1,0) and (0,1) corners of the unit square.

Summary

The components of the deformation tensor are determined directly from geographic space by reading the coordinates of the on-axis corners of a unit square. Those coordinates are column vectors in the matrix. These relationships hold for any 2x2 tensor.

Converted by [Mathematica](#) September 24, 1999

The Deformation Tensor



The Deformation Tensor

The deformation tensor \mathbf{D} is a square matrix that can be reasonably represented in a Mohr circle construction. Here are some definitions relevant to the deformation tensor.

The Stretch of a Line

$$\frac{\text{deformed length}}{\text{undeformed length}} = \frac{l_d}{l_u} = S$$

The Strain of a Line (Elongation)

$$\frac{\text{change in length}}{\text{undeformed length}} = \frac{l_d - l_u}{l_u} = \varepsilon = (1 + S)$$

The strain of a line may also be generally represented by any measure of change in length, including stretch, elongation, and quadratic elongation ($\lambda = S^2$).

The Rotation of a Line

The rotation of a line is a change in the orientation of that line measured as an angle from its orientation in the undeformed state to its orientation in the deformed state. Typically this is done within a fixed reference frame.

Deformation

General deformation is a change in the positions of particles of a body such that initially neighboring particles remain neighbors in the deformed state. Rigid body rotation and translation are special cases where all particles in a body change position in an external reference frame but remain fixed in the internal

reference frame.

Homogeneous Deformation

Homogeneous deformation of a finite region means that parallel and straight lines in the undeformed state remain parallel and straight in the deformed state.

Finite Deformation

A finite deformation results in line length changes and rotations that are visible. In structural geology, longitudinal strains may be more than a few percent and rotations greater than a few degrees. In engineering, finite deformations may be considerably smaller in magnitude than those typically studied by geologists.

Particle Paths and Displacements

A particle path is a trajectory showing the actual movement trail followed by a particle during deformation relative to a particular reference frame. Particle displacement is a vector quantity that represents the difference between two positions on a path.

Progressive Deformation and Deformation

Deformation Tensor Examples

Given a deformation tensor D_{ij}

$$D_{ij} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

and the Mohr circle drawn by plotting

$$\begin{aligned} \mathbf{x}_1 &= (D_{11}, -D_{21}) \\ \mathbf{x}_2 &= (D_{22}, D_{12}) \end{aligned}$$

the position of the circle indicates the style of deformation.

Visualization

Several progressive deformations are illustrated here. Compare the position, magnitude, symmetry and progressive growth of each example. Step through the animations incrementally to relate the tensor components to the Mohr circle. Some of these visualizations are particularly informative if you loop through them back and forth. Choose one of the following links to see an example:

[Simple shear, dextral along \$X_1\$](#)

[Simple shear, sinistral along \$X_1\$](#)

[Simple shear, sinistral along \$X_2\$](#)

[Pure shear along \$X_1\$](#)

[Pure shear along \$X_2\$](#)

[Contraction](#)

[Dilation](#)

[General shear, dextral along \$X_1\$](#)

[General shear](#)

Study the animation for general deformation. What happens when the circle just touches the vertical Mohr axis?

Summary

Progressive animations of the deformation tensor illustrate the similarities in differences between different styles of deformation.

Position Vectors

Given a deformation tensor \mathbf{D}

$$\mathbf{D}_{ij} = \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the geographic coordinates of a particle

$$\mathbf{X}_i = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

the geographic coordinates of the particle in the deformed state are

$$\mathbf{x}_i = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} \cdot \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the standard equations for two-dimensional homogeneous deformation are then

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{11} \mathbf{X}_1 + \mathbf{D}_{12} \mathbf{X}_2 \\ \mathbf{x}_2 &= \mathbf{D}_{21} \mathbf{X}_1 + \mathbf{D}_{22} \mathbf{X}_2 . \end{aligned}$$

These two linear equations describe a position vector \mathbf{x} that gives the coordinates of the particle in the deformed state.

Visualization

This simple animation illustrates the progressive deformation of a unit square. The position vector \mathbf{X} locates the (1,1) corner of the square in the deformed state. When the deformation tensor operates on \mathbf{X} (as well as all other points) the position vector \mathbf{x} tracks the progressive coordinates of the (1,1) point. The lengths of the sides of the resulting rectangle are equal to the magnitudes of the components of the position vector.

Summary

The deformation tensor mathematically changes the positions of particles from one deformation state to another. The new particle positions are described by a position vector that can be deduced from the components of the tensor.

Stretch and Rotation Tensors

A deformation is considered rotational if the principal strain directions rotate in the geographic reference frame. Given a deformation tensor D_{ij}

$$D_{ij} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

and the Mohr circle drawn by plotting

$$\begin{aligned} \mathbf{x}_1 &= (D_{11}, -D_{21}) \\ \mathbf{x}_2 &= (D_{22}, D_{12}) \end{aligned}$$

the deformation can be determined to be rotational or irrotational by inspecting either the off-diagonal components of the tensor or the position of the Mohr circle. In an irrotational deformation, the D_{12} and D_{21} components of the tensor are equal. Thus, the Mohr circle is centered on the horizontal axis of the diagram. Off-axis circles represent rotational deformations.

Visualization

This animation illustrates the graphical decomposition of dextral simple shear into stretch and rotational components. For a shear strain of 1.0

$$D_{ij} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Run the animation. The stretch tensor S_{ij} is found by graphically rotating the D_{ij} circle about the coordinate origin until the centerpoint of the circle lies on the horizontal axis. The coordinates of the \mathbf{x}_1 and \mathbf{x}_2 points of the circle in this position give the appropriate components of the stretch tensor:

$$S_{ij} = \begin{pmatrix} .9 & .45 \\ .45 & 1.3 \end{pmatrix}.$$

(Remember that you must change the sign of D_{21} when going from Mohr space to the geographic space of the tensor.) The rotation tensor R_{ij} is derived from point \mathbf{R} on the diagram. \mathbf{R} marks a unit vector (the light gray arc is a segment of a unit circle) in the direction of the $S_1 - S_2$ line. \mathbf{R} is a point on the circle here giving the tensor

$$R_{ij} = \begin{pmatrix} .9 & .45 \\ -.45 & .9 \end{pmatrix}.$$

You can prove that these two tensors combine to make D_{ij} by taking the dot product of the rotation and stretch tensors.

$$\mathbf{D}_{ij} = \mathbf{R}_{ij} \cdot \mathbf{S}_{ij} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Can you reverse the order of the tensors in the dot product and get the same answer? Will the decomposition of a tensor for general shear (sub-simple shear for example) result in stretch and rotation tensors with symmetry similar to those for simple shear?

Summary

The deformation tensor is composed of a stretch tensor and a rotation tensor. These component tensors can be determined graphically with a Mohr construction by rotating the \mathbf{D}_{ij} circle to the horizontal axis of the diagram.

Stretch

Given a deformation tensor D_{ij}

$$D_{ij} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

and the Mohr circle drawn by plotting

$$\begin{aligned} \mathbf{x}_1 &= (D_{11}, -D_{21}) \\ \mathbf{x}_2 &= (D_{22}, D_{12}) \end{aligned}$$

the principal stretches for the tensor are graphically located at the intersections between the Mohr circle and a line drawn from the Mohr coordinate origin through the centerpoint of the circle. These are the points of minimum (S_2) and maximum (S_1) distance from the origin.

The axial ratio of the strain ellipse as determined from Mohr circle geometry is

$$\frac{S_1}{S_2} = \frac{\sqrt{\left(\frac{D_{11}+D_{22}}{2}\right)^2 + \left(\frac{D_{12}-D_{21}}{2}\right)^2}}{\sqrt{\left(\frac{D_{11}-D_{22}}{2}\right)^2 + \left(\frac{D_{12}+D_{21}}{2}\right)^2}}$$

The values of S_1 and S_2 can be measured directly from the Mohr diagram by rotating those points to the horizontal axis and using the graphical scale.

Visualization

This diagram shows the progressive deformation of a unit square and the growth of a Mohr circle representing that deformation. The deformation is general shear. A marker circle placed on the square graphically records the accumulation of strain. Compare the values of the principal stretches in the Mohr diagram with the numerical values for these quantities and their ratio.

Why do the principal axes of the strain ellipse not have the same orientation in the diagram as the line connecting S_1 and S_2 ? What could you do graphically to make them have the same orientation? How would you go about finding the angle between \mathbf{x}_1 and S_1 in geographic space given the Mohr diagram alone?

Summary

The principal strain axes, in the form of stretches, are easily determined from the Mohr circle for a deformation tensor.

Stretch and Rotation

Given a deformation tensor D_{ij}

$$D_{ij} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

and the Mohr circle drawn by plotting

$$\begin{aligned} \mathbf{x}_1 &= (D_{11}, -D_{21}) \\ \mathbf{x}_2 &= (D_{22}, D_{12}) \end{aligned}$$

the stretch of any line is equal to the length of the line segment connecting a point on the Mohr circle to the origin of the Mohr diagram. The points of minimum (S_2) and maximum (S_1) distance from the origin are the principal stretches of the deformation.

If a deformed unit square is plotted on the Mohr diagram using the same scale as the Mohr circle, it can be demonstrated that a rigid body rotation of the deformed square brings its (1,0) or (0,1) corner into coincidence with the equivalent point on the Mohr circle. In other words, the stretches of these lines are directly equivalent in superposed geographic and Mohr space.

Visualization

This animation has two parts. First, a deformed unit square and a Mohr circle illustrate a progressive general deformation. Both figures are plotted on the same set of axes. Second, the finite deformed square pivots about the coordinate origin until the (1,0) edge is superimposed on the line connecting ($D_{11}, -D_{21}$) and the coordinate origin. The latter line is the stretch of the (1,0) edge. The length of the edge is precisely equivalent to its stretch as measured in Mohr space as expected.

Why do the (0,1) edge and (D_{22}, D_{12}) point not also coincide after the rigid body rotation? Why? Is the rigid body rotation angle in this animation bisected by the horizontal axis of the diagram? If so, will it always be bisected in this way?

Summary

All that separates a deformed line in geographic space and its stretch in Mohr space is a rigid body rotation.

The Infinite Number of Components of the Mohr Circle

Given a deformation tensor D_{ij}

$$D_{ij} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

a Mohr circle is drawn by plotting

$$\begin{aligned} \mathbf{x}_1 &= (D_{11}, -D_{21}) \\ \mathbf{x}_2 &= (D_{22}, D_{12}) \end{aligned}$$

as the end-points of a diameter defining that circle.

It is possible, however, to use any two spatially orthogonal vectors to draw the circle. These two are convenient because they represent the deformation symmetry of a unit square. We typically choose parallel reference axes in geographic and Mohr space for simplicity. Quantities in geographic space may be measured in any of an infinite number of reference frames.

Choosing a different reference frame results in a transformation of the deformation tensor because geographic coordinates of particles vary with the reference frame orientation. There are then an infinite number of tensor components to represent an infinite number of transformed reference frames.

Remarkably, the magnitude and position of a Mohr circle for a particular tensor do not change when new geographic reference axes are selected. These features are invariant properties of the deformation tensor and of the Mohr construction.

Visualization

This animation shows the transformed \mathbf{x}_i' of a general deformation tensor D_{ij} . Here the geographic reference axes go through a complete 180° rotation indicated on the diagram. The \mathbf{x}_i' are transformed coordinates of \mathbf{x}_i . The transformed D_{ij} indicates how the components of the tensor vary during the transformation.

Observe that the chords joining the transformed coordinates remain orthogonal and that the intersection remains at a fixed point called the "pole" of the Mohr diagram. In two instances the \mathbf{x}_i' both fall on one diameter of the Mohr circle. What is the significance, if any, of this coincidence?

Summary

Any Mohr circle, and the tensor from which it is drawn, are represented by an infinite number of tensor components. The circle itself, however, is invariant. Coordinate transformations in geographic space rotate Mohr coordinates around the circle by pivoting around the pole of the circle.

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Andy R. Bobyarchick
Department of Geography
and Earth Sciences
University of North
Carolina at Charlotte
Charlotte, NC 28223
USA

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Converted by [Mathematica](#) September 23, 1999

From Tensors to Mohr Circles

Given a deformation tensor \mathbf{D}

$$\mathbf{D}_{ij} = \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the geographic coordinates of a particle

$$\mathbf{X}_i = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

the geographic coordinates of the particle in the deformed state are

$$\mathbf{x}_i = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} \cdot \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the standard equations for two-dimensional homogeneous deformation are then

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{11} \mathbf{X}_1 + \mathbf{D}_{12} \mathbf{X}_2 \\ \mathbf{x}_2 &= \mathbf{D}_{21} \mathbf{X}_1 + \mathbf{D}_{22} \mathbf{X}_2 . \end{aligned}$$

If the initial position vectors are

$$\begin{aligned} \mathbf{X}_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \mathbf{X}_2 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$

then

$$\mathbf{x}_i = \begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix}$$

and the linear equations for the (1,0) corner of our unit square become

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{11} \\ \mathbf{x}_2 &= \mathbf{D}_{21} \mathbf{X}_1 . \end{aligned}$$

In the same way, the equations for the (0,1) corner become

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{D}_{12} \\ \mathbf{x}_2 &= \mathbf{D}_{22} . \end{aligned}$$

Plotting Convention

The plotting convention used here for drawing Mohr circles from tensor components is (in Mohr space)

$$\begin{aligned} \mathbf{x}_1 &= (\mathbf{D}_{11} , -\mathbf{D}_{21}) \\ \mathbf{x}_2 &= (\mathbf{D}_{22} , \mathbf{D}_{12}) . \end{aligned}$$

This convention is known as a circle of the "First Kind." First Kind conventions have the advantage of preserving similarity of vorticity between Mohr space and geographic space. Carefully note the differences in plotting procedures between geographic figures (such as a deformed unit square) and Mohr figures (such as the Mohr circle.)

Visualization

These static diagrams simply relate primitive deformation tensors to their counterparts in Mohr space.

Summary

Mohr circles are plotted directly from tensors. The positions of the circles reflect the style of deformation while the dimensions of the circles reflect the magnitude.

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The text of the OIMC was created with a variety of tools. Most of the descriptive text was prepared in the math processor *Mathematica* (1999, [Wolfram](#)) and exported directly to HTML format. This procedure has the advantage of typesetting equations internally and then

converting them to GIF files automatically. *BEdit* (1999, [BareBones Software](#)) was used to do additional plain text editing. This application has a powerful HTML editor. Construction of the site was aided by *Dreamweaver* (1999, [Macromedia](#)) and *GoLive* (1999, [Adobe](#)).

Animations were also created with *Mathematica* notebooks and then exported to QuickTime files. Other graphics were prepared or edited with *Canvas* (1998, [Deneba Software](#)), *Illustrator* (1999, [Adobe](#)) and *Photoshop* (1999, [Adobe](#)). All work was done on Macintosh computers, mainly a Power Macintosh 8600 and a Power Macintosh G3.

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